Bye Bye β, Bye Bye

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Bye-Bye, β, Bye-Bye

Enterprise risk management teaches us that reducing or eliminating the deadweight costs of risk can increase shareholder value. Smith and Stulz (1985) is a key paper in the risk management literature and an important turning point – away from the subjective assertion that corporations are inherently risk averse and towards the view that unmanaged risk can destroy firm value.

In their paper, Smith and Stulz demonstrate that risk is costly because bad outcomes are more costly than good outcomes are valuable. More formally, we may observe that the penalty function for risk is convex: if one unit of risk costs shareholders $3MM, two units may cost $7MM. The convexity of the risk penalty has been attributed by Smith and Stulz and others to such things as convex tax schedules, progressive financial distress costs, imperfect contracting, etc.

Many risks may be reduced at little or no cost through hedging (e.g., uncertain fuel costs hedged using energy futures) with the result that the deadweight risk penalty is reduced and firm value is increased. One risk that can be shed at minimal cost is the CAPM systematic risk commonly called β. In this paper we demonstrate that a widely-held firm can increase shareholder value by reducing its β to zero. Until now, the corporate finance literature has been silent on this point, perhaps because "we all know" that firm value cannot be increased simply by buying or selling shares in other companies.

The benefits of hedging away a firm's systematic risk may be amplified when the short position is implemented within a firm's defined benefit plan. Indirectly, this demonstrates that most firms have been destroying value by investing their pension assets in equities.

We begin with an example where risk is symmetric and hedging is costless.

An Example

Consider a firm that consists of a portfolio of one-period projects. The projects are expected to produce an aggregate end of period value of $1000MM. The prospects for the firm are summarized:

Values of Projects and Firm ($ million)					
End of period project value	Probability	Convex risk penalty	Value of firm (unhedged)	Probability (with hedging)	Value of firm (hedged)
1,100	.25	0	1,100	0	
1,000	.50	0	1,000	1	1,000
900	.25	12	888	0	
Expected value →			997		1,000

The projects form a portfolio and the outcome probabilities reflect any diversification benefits to the extent that the portfolio risks are less than perfectly correlated. Hedging, if applied, is done at the portfolio level.

The decision to hedge is based on a comparison of the cost of risk retention ($3MM in the example) to the cost of risk disposal ($0 in the example). If the hedge cost is high or the risk penalty function is less convex, the firm may decide to retain its project risk. Multiple risks will require more sophisticated analysis but, under the value paradigm, the decision criterion will always be cost versus cost.

The Risk Penalty Function

The convex risk penalty function makes firm decisions consistent with a concave (risk-averse) utility function. Thus the observable behavior of a value-motivated firm (as analyzed by Smith and Stulz) may be indistinguishable from the behavior of a risk-averse firm.

Theories about risk-aversion and expected utility decision-making usually postulate a utility function that is concave over its entire domain. This is inconsistent with our intuition about convex risk penalty functions. If we understand financial distress as the weakened ability of the firm to exploit its competitive advantage over a large portfolio of positive NPV projects, we are not surprised that such weakness is initially convex. If project underperformance by $100MM incurs a penalty of $3MM, we may not be surprised to learn that underperformance by $200MM incurs a penalty of $7MM. But, in a limited liability corporation, the penalty function cannot remain convex over its entire domain. It must reach an inflection point and be concave thereafter – simply because the firm runs out of value to lose.

Hedgeable Risks and Project Portfolio Variance

Our simple model addresses project risks that are symmetric (prior to accounting for the penalty) combined with nearly costless hedging opportunities. We would expect the literature to be complete and settled for such a simple model. It seems, however, that the literature has been silent on how one very special risk – systematic or market risk – may enter into this model.

The key to incorporating systematic risk lies in the way in which the risk penalty function responds to project variance. For symmetric distributions, it is increasing and convex as a function of the aggregate project variance. Costless reduction of project variance in the convex zone increases firm value. The function does not make a distinction between systematic and diversifiable risk. Variance is variance, regardless of source.

Hedging Systematic Risk to Minimize the Cost of Retained Risk

By definition a majority of firms have project portfolios with positive systematic risk. Following the CAPM[1], we will use β to designate that risk. We can easily show that minimum variance of the project portfolio occurs when systematic risk is fully hedged. Consider a project portfolio with investment value of $1MM, variance σ_p^2 and β=b. We will hedge this portfolio by shorting $cMM of the market portfolio. We seek the value of c that will minimize the variance of the firm's hedged project portfolio;

$$Var^{uh} = \sigma_p^2$$

$$Var^{h} = \sigma_p^2 + c^2\sigma_m^2 - 2c\rho_{pm}\sigma_p\sigma_m$$

$$\rho_{pm} = b\frac{\sigma_m}{\sigma_p}$$

$$Var^{h} = \sigma_p^2 + c^2\sigma_m^2 - 2cb\sigma_m^2$$

$$\frac{\partial Var^h}{\partial c} = 2c\sigma_m^2 - 2b\sigma_m^2$$

$$\frac{\partial^2 Var^h}{\partial^2 c} = 2\sigma_m^2 > 0$$

$$\frac{\partial Var^h}{\partial c} = 0 \Rightarrow c = b$$

where Var^{uh} and Var^{h} represent the project variance without and with hedging; σ_m^2 is the variance of the market portfolio; ρ_{pm} is the correlation coefficient between the project and market portfolios; the next to last expression demonstrates that we have found a minimum variance; and the last expression shows that the minimum variance is achieved when the (negative) hedge ratio equals the β of the unhedged project portfolio. In other words, minimum variance is achieved when the firm shorts its own β entirely. We note that, for those unusual firms with negative β, the hedge consists of acquiring β.

The implication is clear: firms facing a convex risk penalty function should hedge the systematic risk in their project portfolios. Their shareholders may wish to reverse the transaction in order to preserve their own portfolio risk level. The net gain from such transactions is measured by the reduction in deadweight costs attributable to the risk penalty.

Measuring Financial Distress

Almeida & Philippon (2005) measure financial distress costs for publicly traded companies. They point out that the (negative) value of financial distress has a systematic component – financial distress is more likely to occur when capital markets are weak and less likely when they are strong. Thus the absolute value of financial distress has negative β and it should be computed using a below risk-free rate of discount. The prevailing literature has ignored this systematic risk and financial distress is commonly discounted using WACC or risk-free rates.

The negative β in a firm's financial distress may be seen as an echo or mirror of the positive β in its projects. When projects do badly, financial distress costs are incurred and positive-β projects do badly when markets are weak. Thus a firm that hedges its project β will lower its ex-ante financial distress cost and increase shareholder value.

The Pension Gambit

Shorting β may be readily accomplished in the capital markets using futures, swaps or ETF's. But shorting β on the firm's balance sheet can create liquidity and cash flow difficulties. Firms that sponsor defined benefit (DB) plans may more easily short β in those plans. Serendipitously, shorting inside a DB plan produces additional value for the sponsor's investors. Gold and Hudson (2003) demonstrate that DB plans that divest themselves of β increase after-tax firm value by approximately 30% of the value of the equities they sell. Gold (2001) shows that this ratio extends as well when the plan establishes short equity positions. Neither of these papers accounts for the financial distress gains articulated here and thus DB plans provide a second source of

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[1] The analysis may be easily extended to more general asset pricing models as long as they imply a risk premium whose sign matches the covariance between the security and the market.

value for positive β firms that elect to hedge. Negative β firms should acquire equity investments on their balance sheets and not in their DB plans.

Future Directions

The deliberately simple model articulated herein is designed to highlight the core insight that firms facing convex risk penalties may enhance shareholder value by disposing of systematic risk. We have looked at symmetric risks and costless hedging. Gold (2006) extends this exploration to include asymmetric risk (especially one-sided insurance-type exposures) and costly hedging. The penalty functions herein are convex in outcome and convex with respect to symmetric risk. The implication to reduce systematic risk may also be derived from risk distributions that induce convexity even when penalties are linear (or mildly concave). The Almeida and Philippon findings are an example of this phenomenon. They show that a fixed singular risk penalty that is more likely to occur in weak markets has properties and implications similar to those we have identified here as attributable to convex penalties.

Although we have used the CAPM and β to illustrate the role of systematic risk herein, similar and more general results may be derived using the pricing kernel concept that is common to many asset pricing models.

There is an analogy between the firm that zeroes its own β by hedging and the activities of α-seeking asset managers, absolute return managers, and hedge funds. In a sense, the firm that hedges has become an α-firm. If it is truly able to exploit and grow its franchise value, it will provide value to its investors that is independent of market conditions generally.

But just as the pursuit of α raises the obvious question: how can everybody earn positive α in a zero sum game, hedging of β by firms raises the question: what if every firm wanted to do this? Who will hold systematic risk if the corporations that generate it will not? This equilibrium question will have to be the subject of further and future research.

References

Almeida and Philippon (2005)

Gold (2001)

Gold (2006)

Gold and Hudson (2003)

Stulz and Smith (1985)