Fair Value of Liabilities: The Financial Economics Perspective

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Abstract

In this paper we present the fundamental approaches of financial economics to valuation. Three methods are demonstrated by which financial economists account for risk. We illustrate how these methods relate to one another and how they can be applied in the valuation of risky corporate bonds, guaranteed investment contracts (GICs) with and without interest rate contingencies, and whole life insurance. Next, we discuss how these models treat orthogonal risks, such as the kind often covered by insurance contracts. Demand side and supply side diversification are treated, and liquidity risk is then considered. We conclude with a summary of the benefits of decomposition and transparency.

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1. Introduction

There has been considerable discussion of a variety of issues related to fair value in the actuarial literature, in conferences, and among individuals interested in this topic. Unfortunately we seem to be failing to communicate due in part to a lack of a consistent paradigm and objectives. In this paper we present a few concepts that we hope will be of use in the broader discussion of fair value of liabilities.

The financial economics paradigm is not the only perspective that is relevant to the fair value debate. Obviously there is an entire body of literature and practice in accounting, actuarial science, taxation, and regulation that will bear sway in the fair value debate. It is important, however, that the final form of fair value accounting does not deviate from well-established valuation principles that are tested by the entire world capital markets on a daily basis. Those valuation principles that emerge from the finance and economics literature and practice that do not hold up to empirical testing are rejected. That rejection often takes the form of market failure by those who implement unsound strategies.

The American Academy of Actuaries, the International Actuarial Association, the FASB, the IASB, the SEC, and others have organized committees and task forces and sponsored symposia discussing issues relating to the valuation of insurance liabilities. The goal of this paper is to review fundamental approaches to valuation from a financial economics perspective. Then we will discuss the treatment of default risk, the pricing of risks that are orthogonal to the market, and the impact of illiquidity on insurance liability valuation.

2. Fundamental Approaches to Valuation

In the absence of observable market prices, there are at least three theoretically correct methods for estimating the value of a series of (potentially risky) future cash flows. (1) Discount the true probability-weighted future cash flows using discount rates that are the sum of a risk-free rate and a risk premium. (2) Modify the probabilities of the risky future cash flows to account for risk and discount at risk-free interest rates. (3) Modify the risky cash flows to account for risk and discount

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at risk-free rates. We will discuss each briefly in the form of a one-period example.[1]

Consider a security with price S that will pay either S_u or S_d in one year. We can apply the three methods of valuation as follows. First,

$$S = \frac{pS_u + (1 - p)S_d}{1 + r + \lambda\sigma_s}, \quad (1)$$

where r is the one-year risk-free rate, p is the "true" probability of the payoff being S_u, λ is the market price of risk associated with the uncertainty about the security's payoff, and σ_S is a volatility parameter associated with the uncertainty of the security's payoff.[2] Second,

$$S = \frac{\pi S_u + (1 - \pi)S_d}{1 + r}, \quad (2)$$

where $\pi = p - \lambda\sqrt{p(1-p)}$ is the risk-neutral (martingale) probability. Or, third,

$$S = \frac{[pS_u + (1 - p)S_d] - Z}{1 + r}, \quad (3)$$

where Z is a quantity that makes the numerator of Equation (3) equal to the certainty equivalent of the risky expected payoff in the numerator of Equation (1).

In order to illustrate how pricing with martingale probabilities compares to pricing with the "true" probabilities or using a certainty equivalent, consider the problem of valuing a simple one-year interest-rate-contingent claim. This claim will pay \$110 if the short rate goes up and \$90 if the short rate goes down. This claim can be valued using the "true" probability, $p = 0.51$, and a risk-adjusted discount rate. The risk-adjusted discount rate is

$$r + \lambda\sigma_s = 0.0520995,$$

where $\lambda = 0.02$ and $\sigma_s = 0.104979$. Thus, this security's value is

$$[p\ \$110 + (1 - p)\ \$90]/(1 + r + \lambda\sigma_s) = \$95.24.$$

Similarly, this security can be valued using the martingale probability, $\pi = 0.5$, and discounting at the risk-free rate, $r = 0.05$:

$$[\pi\ \$110 + (1 - \pi)\ \$90]/(1 + r) = \$95.24.$$

Finally, using the certainty equivalent approach with $Z = 0.2$, the value would be

$$[p\ \$110 + (1 - p)\ \$90 - 0.2]/(1 + r) = \$95.24.$$

The conclusion is that the valuation process can account for risk, either by using the "true" probabilities and discounting by a risk-adjusted discount rate, or through converting the "true" probabilities into martingale probabilities and discounting by the risk-free rate, or by adjusting the cash flows to their certainty equivalent levels and discounting at the risk-free rate.[3]

Each of these three approaches is theoretically correct. Practical considerations dictate the choice among the three approaches. Equation (1) is the traditional discounted cash flow model. It is most often used for capital budgeting and net present value type of analysis. It is also the traditional method of choice for nontraded or thinly traded securities. Equation (2) is a one-period lattice version of the option pricing model. The existence of the martingale probabilities arises from the ability to create a hedge portfolio in a complete market. The hedge portfolio exactly replicates the cash flows of the security under consideration. In fact, the ability to create a hedge portfolio is synonymous with markets being complete. This approach is used when pricing

[1] This example is drawn from Babbel and Merrill (1996), *Valuation of Interest-Sensitive Financial Instruments*, pp. 43–44. Extension to the multiperiod case is covered in the monograph. Here,

$$\sigma_s^2 = [(S_u - S_d)/S]^2 p(1 - p).$$

[2] The market price of risk is the equilibrium excess reward to risk ratio, where μ_S is the expected return and σ_S is the standard deviation of return for the security, S. In equilibrium the reward-to-risk ratio is constant for all securities. In a CAPM framework (see Section 4.1) λ would be defined with β in the denominator. In a multifactor setting there would be a market price of risk for each stochastic factor.

[3] Notice that if there were no risk and the security, S, were to pay with certainty an amount equal to the expected payoff in the example, $[p\ \$110 + (1 - p)\ \$90] = \$100.20$, then the value of S would be $\$100.20/(1 + r) = \$95.43$. This result can be seen in Equation (1) because $\sigma_s = 0$ when the cash flow to S is certain. In Equation (2) the numerator is replaced by $[\pi\ \$100.20 + (1 - \pi)\ \$100.20]$, and in Equation (3), when there is no risk, $Z = 0$. Because a nonzero value of Z is subjective (utility dependent), it is useful to note that all three approaches collapse to the same value when considering risk-free securities.

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interest-sensitive financial instruments and other derivatives in a complete market. Equation (3), the certainty equivalent method, is not often used because the certainty equivalent adjustment, Z, is dependent on the form of a utility function. However, it has been successfully used in capital budgeting problems.

There are examples where the option pricing model[4] has been successfully applied to thinly traded securities. Probably the most prominent are certain kinds of mortgage-backed securities (MBSs). The underlying prepayment risk was not actively traded until the creation of MBSs. Uncertainty surrounding prepayment risk, and perhaps other risks not being modeled in the contingent cash flows, were, and still are, accounted for using an option-adjusted spread (OAS). In essence, the prepayment and other risks are modeled and priced using the OAS as an increment to the risk premium of Equation (1), and the interest rate contingencies are priced using a risk-neutral measure over future possible interest rates as in Equation (2). The OAS can be thought of as a fudge factor added to the discount process that reconciles the models with the market.[5] Over time as improved prepayment models left smaller residuals and active trading emerged, the OAS shrunk drastically on vanilla MBSs when valued using properly calibrated, adequate models. This is an important example of how transparency and market mechanisms can improve market efficiency.

3. Default Risk

Another example of an application of the option pricing model to thinly traded assets is the pricing of corporate bonds. Merton (1974) as well as Black and Scholes (1973) suggested that corporate securities could be viewed as options on the underlying assets of the company.[6] The underlying assets include plant and equipment, franchise value, customer relationships, etc. These parts of the asset value are difficult to observe and price. Nonetheless, the model still has been used successfully in pricing credit derivatives. The inability to observe the value of assets is less of a concern for insurance liabilities, where the vast majority of assets are financial and easily observed.

Consider a simple nonfinancial company with equity holders and a single bond issuance. Note that the bondholders are entitled to the value of the assets up to the face amount of the debt and that the equity holders are entitled to the value of the assets in excess of that amount. This means that we can view equity as a call option on the assets with a strike price equal to the face value of the debt. For a zero coupon bond, in a world of constant interest rates, the value of equity is given by the Black-Scholes call option formula. Extensions for coupon bonds have also been derived. The value of the bond is given by subtracting the equity call option from the underlying assets. Thus, the bondholders are described as owning the assets and selling a call option to equity holders.

Recall the Black-Scholes call option formula

$$C = AN(d_1) - Xe^{-rT}N(d_2),$$

where

$$d_1 = \frac{\ln(A/X) + (r + \sigma^2/2)/T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

and where

C = call option value = value of equity in the Merton model
A = current asset value of the company
$N(d)$ = standard normal density evaluated at d
X = exercise price = face value of debt

[4] Other names applied to this model include the martingale measure, risk-neutral probability, or hedging model.

[5] OAS is included in MBS models because of suboptimal exercise of the prepayment option. Financial economists typically model prepayment behavior with a flexible functional form, but the function never fits the behavior exactly; rather, there is always noise surrounding the estimates. Even though the deviations may be independent and symmetric, the impact of the deviations is asymmetric to the investor. For example, surprise prepayment when interest rates are low leads to greater cost because of worsened reinvestment opportunities than when interest rates are high. Moreover, OAS arises because of the asymmetric costs of modeling error.

[6] These models are too simplistic for pricing corporate bonds in practice. This model is, however, sufficient to illustrate the key concepts that are relevant to this discussion. Extensions that accommodate the complexities of these bonds include Duffee (1999), Duffie and Lando (2000), Duffie and Singleton (1999), Jarrow and Turnbull (1995), and Kim, Ramaswamy, and Sundaresan (1993).

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r = risk-free rate
T = time to maturity for the option and
σ = standard deviation of the annualized continuously compounded rate of return on the assets.

Then the value of the bond is $A - C$.

Three key observations can be made at this point. First, the bond value converges to a risk-free bond value as the asset value of the company increases. Second, the value of the bond decreases as the volatility of assets increases. And, third, the expected return on assets is not an explicit component of the value of the bond. We will comment on each point in turn.

An increase in asset value increases both the value of equity and the value of debt, up to a limit. The most that the bond can be worth at maturity is X, the face value of the debt. As the value of assets increases, the value of the equity converges to $C = A - Xe^{-rT}$. This can be seen by observing that as A grows large relative to X, d_1 and d_2 increase and the call option (equity) value increases toward an upper limit of $C = A - Xe^{-rT}$. Then the value of the bond is $A - C = Xe^{-rT}$. Thus, for very large asset values, the bond is risk free, and the price of the bond is the promised cash flow discounted at the risk-free rate. Notice that this result holds for relatively conservative assets with a low standard deviation or for very risky assets with a large standard deviation. For any given risk level (standard deviation of assets) the bond will be risk free for a sufficiently large asset level.

The second point deals with volatility. It is a standard result in option pricing that an increase in volatility increases the value of an option. This can be seen by taking the derivative of the option pricing formula with respect to σ. Therefore, all else being equal, the value of the bond decreases when volatility increases. This is an intuitive result. Higher volatility in the assets leads to a greater probability of the firm defaulting and the bond holders receiving the assets of the firm as partial payment of their claim. Thus, our first point does not violate the simple intuition of this second point.

Finally, many students find it troubling that the expected return on assets is not an explicit component of the equity or bond value. Although the option pricing formula involves discounting at the risk-free rate, the relationship between the martingale probabilities inherent in the option pricing formula and the "true" probabilities depends on the risky return on assets. Recall Equation (2): the martingale probability, π, is a function of the "true" probability and the market price of the underlying risk. The same intuition holds in the more complex Black-Scholes option pricing formula. The market price of the asset risk of the company enters into the relationship between the martingale measure, $N(d)$ in the option pricing formula, and the "true" probability density.

There is an alternative representation of the value of a corporate bond in the option pricing framework. Recall the put-call parity relationship

$$P = C + Xe^{-rT} - A,$$

where P is the price of a put option written on the same assets, A, having the same strike price, X, and the same time to maturity, T, as the call option, C. The put-call parity formula can be rewritten as

$$A - C = Xe^{-rT} - P. \quad (4)$$

Notice that the left-hand side of Equation (4) is the value of the bond, as described above. The right-hand side of Equation (4) is the price of a risk-free bond minus a put option. Thus, a corporate bond value can be decomposed into a risk-free bond and a put option on the assets of the firm. For convenience we will refer to the value of the bond cash flows, discounted at risk-free Treasury rates, as the *synthetic Treasury value* of the bond. Thus, the decomposition involves two terms: the synthetic Treasury value of the bond and the put option.[7] This is a useful decomposition, as we can now observe the relative impact of interest rate changes and credit quality changes. Interest rate changes will impact both terms, but the price of the risk-free bond will capture only the pure time value of money. When the credit worthiness of the firm changes, that will be captured by the put option value.

It is important to note that the option pricing approach differs from simply discounting liability cash flows at Treasury rates and calling the re-

[7] In Merton's original derivation of this model the only risk captured by the option was default risk. In an insurance liability application it would need to capture other risks such as mortality, casualty, expense, or illiquidity risks.

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sulting present value the fair value of liabilities. As has been pointed out repeatedly and forcefully, there must be some accounting for risk. The accounting for risk is done properly in our decomposition approach. Notice, though, that simply using the asset portfolio return as a discount rate would be a mistake. The asset portfolio return is not the key to the risk in the liabilities. The keys are the degree of overcapitalization $(A - X)$ and the volatility of asset returns.

The insights from the Black-Scholes option pricing model are applicable, strictly speaking, only to corporate debt with a fixed promised payment, X. When debt payments are interest sensitive, in the sense that they are linked, either explicitly or implicitly, to interest rate levels and/or their evolution over time, we must resort to multiperiod variations of valuation formulas (1), (2), or (3), as described more fully in Babbel and Merrill (1996, 1998).

3.1 A Bullet GIC

The valuation principles from the previous section may be illustrated in an insurance context by focusing on the simplest insurance liability with a fixed, promised payment at the end of the period—the bullet GIC. In its simplest form the bullet GIC is little more than a zero coupon bond. The fair value of the bullet GIC could be determined using any of the valuation approaches discussed above. There are several reasons, however, that we suggest it should be valued as a risk-free zero coupon bond minus a put option. As stated earlier, no correctly implemented valuation approach is more theoretically correct than any other correctly implemented valuation approach. The choice of valuation methodology is often driven by practical considerations.

If the bullet GIC were the only type of liability issued by an insurance company, we could just calculate the market value in the most convenient way possible. We could simply look to the secondary market, thin though it might be, and price accordingly. Alternatively, we might look to the creditworthiness of the issuer and add a spread to Treasury STRIP[8] rates to discount the promised cash flow from the bullet GIC. The liabilities of an insurer, however, are much more complex than a simple bullet GIC. It is when we turn to the more complex liabilities that the decomposition into a risk-free liability and a risk-based put option becomes particularly desirable.

3.2 A Bullet GIC with Interest Rate Contingencies

Consider a stylized window bullet GIC as a very simple extension. The window gives an investor the option either to buy a GIC now at a set interest rate, or to wait some amount of time and buy a GIC that offers either prevailing market rates or a guaranteed minimum rate, whichever is greater. The decision to take advantage of the window feature of the GIC will be made based on current market interest rates at the inception of the window GIC and at the time of the closing of the window.[9] Thus, the window GIC has interest rate contingency in its potential cash flows. In other words, the possible cash flows, and their timing, depend upon the evolution of market interest rates during the open window period. In terms of valuation, we can decompose the value of the window GIC into a default risk-free window GIC and a default risk-based put option value, as in Equation (4). The difference between this application and Equation (4) is that the default risk-free window GIC value would not be Xe^{-rT}, but would be

$$E_r[X_T(r)e^{-rT}],$$

where E_r is the risk-neutral (martingale) expectation over possible interest rates, and $X_T(r)$ represents the interest-rate-contingent cash flows to the window GIC.[10] The default risk-based put option value, P, in Equation (4) would capture the default risk inherent in the issuer of this putable GIC.

A simple two-period numerical example will help to illustrate the valuation of interest rate contingencies. For simplicity we will assume that each period is equal to one year. The first step in the example is to describe the interest rate environment. A two-year interest rate lattice will be used to describe the possible evolution of Treasury short rates (one-year spot rates of interest). Assume that the current one-year T-bill rate is 5%

8 STRIP is the acronym for Separate Trading of Registered Interest and Principal Securities.

9 This example assumes rational behavior on the part of the investor. Later we discuss how to treat suboptimal utilization of policy options.

10 See Babbel and Merrill (1998) for a more complete exposition on interest-rate-contingent valuation in this setting.

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and that the one-year T-bill rate in one year could either rise to 6.17% or fall to 4.05%. This assumption is consistent with a simple multiplicative model with a volatility parameter of 0.234.[11] Thus, our Treasury short rate (T-bill rate) lattice is

$$5.00\% \begin{array}{l} \nearrow 6.17\% \\ \searrow 4.05\% \end{array}$$

The second step in this example is to figure out the two-year spot rate that is consistent with this model of the interest rate environment. The spot rate is found by pricing a two-year zero coupon bond and finding its yield. The pricing lattice, assuming again a risk-neutral probability of an up movement in interest rates of 0.5, is

$$\$906{,}164.20 \begin{array}{l} \nearrow \$941{,}885.70 \begin{array}{l} \nearrow \$1{,}000{,}000 \\ \searrow \$1{,}000{,}000 \end{array} \\ \searrow \$961{,}059.20 \begin{array}{l} \nearrow \$1{,}000{,}000 \\ \searrow \$1{,}000{,}000 \end{array} \end{array}$$

Given this price, the two-year spot rate is

$$\sqrt{\frac{\$1{,}000{,}000}{\$906{,}164.20}} - 1 = 5.05\%.$$

As a baseline, assuming no default risk, $1,000,000 invested in a simple two-year bullet GIC would offer an annual yield of 5.05% and a terminal value of $1,103,550. This is also the expected payoff to a two-year strategy of rolling over an investment in T-bills.

The third step to this example is to value the interest-rate-contingent cash flows of the window GIC. Consider a default-free window GIC that offers the same market rate of 5.05% for two years but also offers the buyer the window option to wait one year and then put money into the GIC with a guaranteed rate of 4.9% in one year for the one remaining year. Because of the window option, this GIC must be valued in the lattice framework.[12]

If the GIC investor chooses to put money into the window GIC today, then they receive the same $1,103,500 as if they had invested in the simple two-year GIC above. If, however, the investor chooses to wait a year to put money into the window GIC, then they can buy $1,000,000 of T-bills yielding 5% for one year and then put the proceeds into the window GIC at the end of the first year for the second year. At the end of the first year the investor will put $1,050,000 into the window GIC either at market rates or at the guaranteed rate of 4.9%, whichever is greater. If the market rate rises to 6.17%, the investor will put their money into the equivalent of a simple GIC with a yield of 6.17% and receive $1,114,785 at the end of the second year. If, however, market rates fall to 4.05%, the investor will exercise the window option and put their money into the window GIC at the guaranteed rate of 4.9% and receive $1,101,450 at the end of the second year.

The investor's choice, then, is to commit to the two-year spot rate of 5.05% and receive $1,103,500 for certain, or to face the equivalent of a 50/50 lottery over receiving either 5% in year 1 and 6.17% in year 2, leading to a terminal value of $1,114,785, or receiving 5% in year 1 and the guaranteed rate of 4.9% percent in year 2, leading to a terminal value of $1,101,450. Using the lattice valuation methodology we can put a fair market value on the interest-rate-contingent cash flows associated with the window option of the window GIC. The lattice for discounting the two possible paths for the window option is

$$\$1{,}004{,}076 \begin{array}{l} \nearrow \$1{,}050{,}000 \rightarrow \$1{,}114{,}785 \\ \searrow \$1{,}058{,}559 \rightarrow \$1{,}101{,}450 \end{array}$$

This represents an arbitrage opportunity for the investor. The fair value of the window option cash flows is $1,004,076 even though these cash flows can be obtained by spending only $1,000,000. Thus, the investor realizes an instant profit of $4,076 at the beginning of year 1. This profit comes at the expense of the insurance company issuing the window GIC. The arbitrage opportunity would disappear if the insurance company required the investor to pay a fee of $4,076 for the right to wait and exercise the window option. Then the investor would be indifferent between putting money into the window GIC at the start of year 1 and paying the fee to exercise the window option.

[11] The model used in this example is the simple multiplicative model used in chapter 3 of Babbel and Merrill (1996). The lattice valuation techniques used in this example are explained in more detail there.

[12] We recognize that real-life window GICs do not offer one-year windows to lock in an interest rate. This stylized example is offered to illustrate the valuation issues that would apply even for very short window options on GICs.

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Remember that this example illustrates the determination of the synthetic Treasury value of the window GIC liability, $E_r[X_T(r)e^{-rT}]$. The default put option value, P, will depend on the actual overcapitalization, $A - E_r[X_T(r)e^{-rT}]$, and on the asset volatility of the insurer, as described above. It is important to understand that in using the decomposition that we propose, all interest rate contingencies are evaluated and incorporated into the synthetic Treasury portion of the liability value. This portion of the valuation can be done using the risk-neutral valuation approach illustrated by Equation (2). Then default risk is evaluated and reported as the default put option illustrated in Equation (4).

As we have stated before, this is not the only correct approach to valuation; rather, it is the approach that we feel has the greatest benefit for reporting purposes. It is the decomposition that allows (or even emphasizes) the source of changes in value because the impact of interest rate risk is separated from default risk. Failure to recognize the interest rate contingency can result in unprofitable business to issuers of these securities.

3.3 Whole Life Insurance

Even more complex than the GICs discussed above would be an insurance company liability such as whole life insurance. Whole life includes not only interest rate contingencies, but also other risks, specifically, mortality risk. Consider a very simple form of whole life insurance that offers protection against mortality risk with a level annual premium for the duration of a policyholder's life. The policy offers only the right to surrender and take away any cash value in the policy but no policy loan, settlement options, or other typical policy options. In this case the synthetic Treasury value of the liability analogous to the Xe^{-rT} term in Equation (4) would be

$$E_r\{E_{y|r}[X_T(r, y)e^{-rT}]\},$$

where, as before, E_r is the risk-neutral expectation over the distribution of interest rates, but now mortality risk, y, must be considered. The interior expectation uses true mortality probabilities and is taken over possible mortality outcomes *given* an interest rate level. Then the risk--neutral expectation is taken over possible interest rate levels. As with the examples above, default risk is treated in the default put option value analogous to Equation (4). The interest rate contingencies are valued in the synthetic Treasury value portion of the liability valuation. So, ignoring default risk for now, the possible cash flows are potentially a function of both interest rates and mortality. If mortality risk is orthogonal to interest rate fluctuations, the surrender decision is the only source of interest rate contingency.[13] Valuation of the surrender decision is analogous to valuation of the prepayment decision for mortgage-backed securities.

Figure 1 illustrates the valuation process for a liability like whole life. The interest rate lattice is used just as usual for projecting possible future interest rates and for discounting possible future interest-rate-contingent cash flows. The surrender decision is illustrated in the panels that relate to two illustrative nodes on the lattice. The spread between market rates (the interest rate at the node of the lattice) and the crediting rate minus surrender costs drives the policy surrender decision. The two lower panels illustrate the spread of surrender behavior as a function of interest rate spreads and show a curve fit to the surrender data. The distribution of policy surrender behavior around the curve varies across spread levels. In the valuation, though, it is the fitted curve that is used to project interest-rate-contingent-cash flows that are discounted through the lattice.

As long as mortality risk is orthogonal (uncorrelated) to interest rate risk, and fully diversifiable, this approach is simple: just replace any stochastic representation of mortality-dependent cash flows by their mean. We will discuss the pricing of orthogonal risks in the next section.

4. ORTHOGONAL RISKS

More general insurance liabilities pose even greater challenges for valuation. There are under-

[13] Unlike the case of Equation (4), where the insurer has an option to default, the whole life policy grants an option (i.e., to surrender or persist) to the policyholder. Therefore, the value of the liability is increased by the value of this put, not decreased as in Equation (4). Indeed, the put in Equation (4) is not shown with interest rate contingencies built in.

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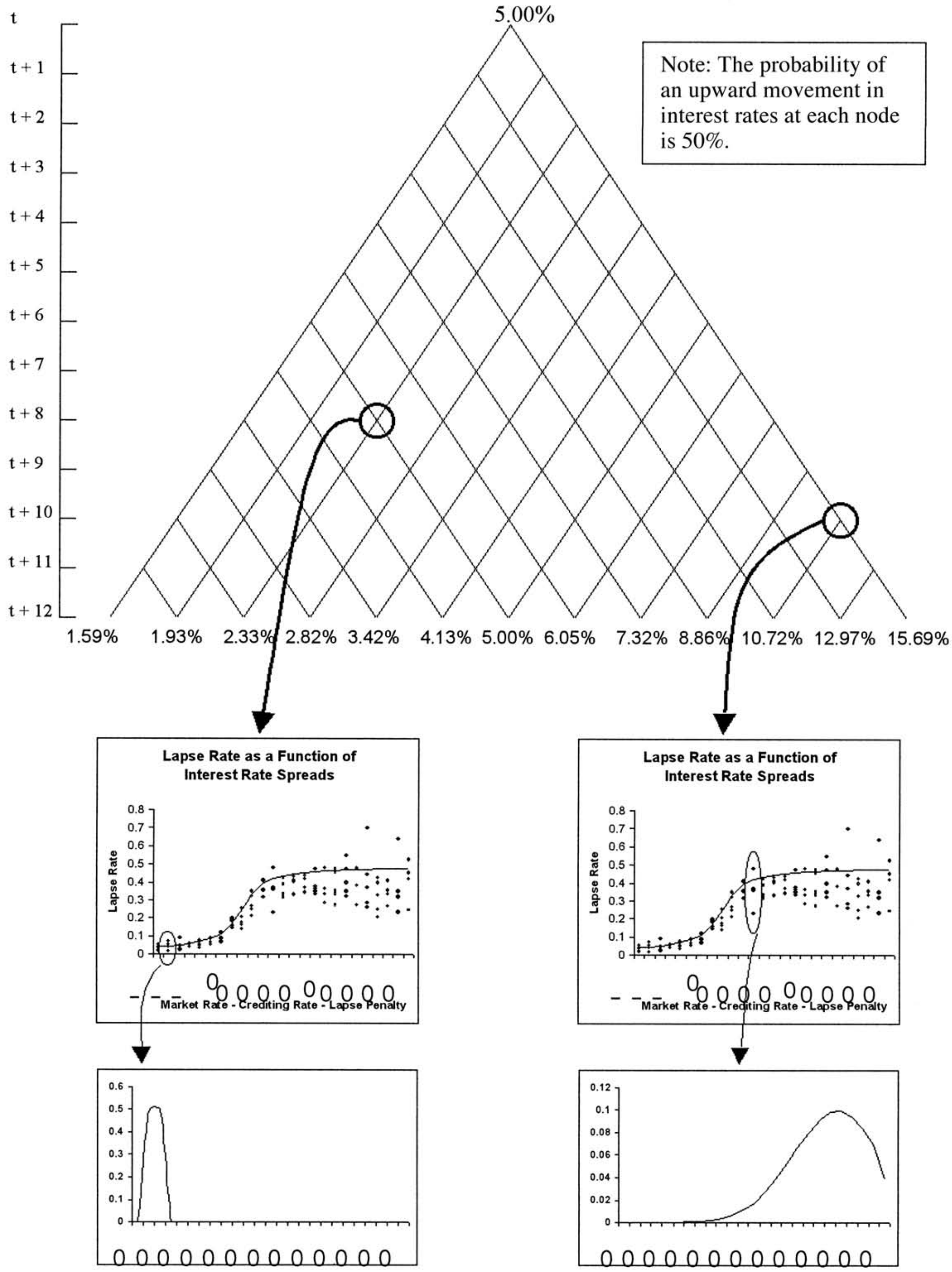
Figure 1
Uncertainty Surrounding Interest Rate Contingent Cash Flows



writing risks, mortality or casualty risks, operating and expense risks, nonoptimal use by policyholders of options granted in the policy, etc. However, as shown above and discussed in Babbel and Merrill (1999), these risks can be incorporated into a model with interest rate risks. If the other risks are orthogonal (uncorrelated) to interest rate or market risks, they are not directly

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"priced."[14] If, however, they are not orthogonal or not diversifiable, then they must be priced in the model. It is common, as in the case of mortgage-backed securities, to add an option-adjusted spread (which can be positive or negative) into an interest-rate-contingent valuation in order to account for non–interest rate risk that is not diversifiable or is not completely orthogonal to interest rate or market risk.

4.1 Naturally Orthogonal Risks and Acquired Beta

Several equilibrium pricing models have appeared in the financial economics literature. Common among most of these models is that orthogonal risks do not command a "market premium," whereas systematic risks do. In this section we will consider one of the earliest of these equilibrium pricing models, the capital asset pricing model (CAPM: Sharpe 1964), to illustrate these concepts. Thus, we are assuming a world consistent with the CAPM assumptions. We will then contrast these pricing insights from theory against what we observe in practice and discuss how orthogonal risk pricing should be reported.

CAPM derives the following representation for the one-period expected return on any risky asset, i:

$$E(r_i) = r_f + \beta_{i,m}[E(r_m) - r_f],$$

where r_i and r_m represent the random returns on security i and the market portfolio, respectively, and r_f represents the certain (risk-free) rate of return for the period.

The term $[E(r_m) - r_f]$, known as the market equity risk premium, is positive because investors are risk averse and will prefer a certain return to any risky return with lesser or equal expectation.

$\beta_{i,m}$ is a scaled measure of the correlation between the market rate of return and the security return. A positive value for $\beta_{i,m}$ indicates that this correlation is positive and implies that the expected security return is in excess of the risk-free rate. Stocks whose returns tend to move with the aggregate market carry positive equity premiums because these securities are likely to have poor returns when the overall market is weak. A negative value for $\beta_{i,m}$ implies a negative equity premium; such securities may provide their best returns when the market is depressed and such returns are most valued.

A very interesting special case occurs when there is no correlation between a security's return and the market return. Such zero-beta securities carry none of the systemic risk of the market. The CAPM says that the expected return on a zero-beta risky asset is equal to the risk-free return.[15] The risks embedded in such returns are alternatively described as "orthogonal" (meaning uncorrelated with market returns) or "nonsystematic" or "idiosyncratic." A related observation is that "the market does not reward nonsystematic risk" or "the market does not price orthogonal risks."

The intuition that supports these observations is that rational risk-averse investors evaluate securities for their contribution to risks and returns of the portfolio rather than as individual securities. Investors are choosing preferred distributions of risky returns at the portfolio or market level. Portfolios with identical return distributions must be equally valued even if the actual returns may differ between such portfolios. The marginal inclusion of zero-beta securities may alter ex post returns but do not alter the ex ante distribution of well-diversified portfolios.

Most securities have positive beta (since the market portfolio includes all securities), but, from time to time, energy stocks and various commodity stocks (e.g., gold mines) have had negative beta since they tend to prosper when the rest of the economy suffers. But what does a zero-beta security look like? The most prominent case is the default-free zero coupon bond for the period contemporaneous with the time period over which the expectations are taken in the CAPM. Because the return on this instrument has no uncertainty, it has no correlation with the stochastic market return and thus is zero-beta. Con-

[14] Financial economists use the term "priced" as an actuary might say "charged for" in a premium calculation or "accounted for" when included in an insurance liability. It specifically means that the premium or liability includes an amount above and beyond the mean value of the random variable.

[15] The CAPM may be derived under the assumption of a multivariate normal distribution of returns. When distributions are asymmetric (e.g., skewed), we should consider higher moments (unless utility is quadratic). Under an extended CAPM, informational independence (i.e., all co-moments equal to zero) is sufficient to imply a risk-free expected return. See Dybvig and Ingersoll (1982).

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sistently, it offers the zero-beta, or risk-free, rate of return.

Another example of a zero-beta security is an orthogonal risk such as the outcome of a coin toss where the amount wagered is minuscule in relation to size of the bettor's portfolio wealth. The risk is orthogonal because the coin is no more likely to land on heads or tails in an up or down market. A similar risk is a tiny bet on whether or not it will rain this afternoon in Arizona. So too is a small partial indemnification that I may offer in the event that your house burns to the ground during the next year. If you give me a penny today and the chance that your house burns is one in one hundred, I will promise you one dollar plus the risk-free return thereon next year if your house burns down.

These risks are inherently zero-beta because of a lack of correlation with the rest of the market portfolio (which, in its most general sense, is the world's aggregate wealth portfolio). These risks can be contrasted with the risks inherent in the ownership of General Electric stock. GE stock will exhibit a positive correlation with the non-GE market because GE and the rest of the market respond to common factors (e.g., inflation, labor supply, interest rates). This is not the case for the coin toss, rain in Arizona, or your house fire. If, after a year of isolation, I am told the return on the market, I will be able to form an updated estimate of GE's return, but I will have no new information about your house.

But why have we used words like minuscule, tiny, and small? Each of the orthogonal bets that we have described becomes a security when the wager is made. In the case of the coin toss, the game is zero sum and the market portfolio is unaffected. This is seemingly so with respect to the house as well. The insured "wins" when the insurer loses, and vice versa. But the insurance case indicates that the value of the house has been in the market portfolio all along, and thus the loss of the house must, in some small way, be correlated with the very broadly defined market. The insurance does not alter the market portfolio because it is a zero sum game, but it does highlight the relationship between the pre-insurance risk and the market.

What can we say about the beta of a risk that is independent of the market except for its own inclusion in the market? Once we know about this risk (and we will treat it as though it were a security, s), we must redefine the market portfolio as the sum of the market we thought we had (i.e., m) and s. Our market portfolio is now m plus s. Compute $\beta_{s,m}$ where s is not part of m, and then compute the "acquired" $\beta_{s,m+s}$ where s is included in the market:

$$\beta_{s,m} = \frac{\text{cov}(s,m)}{\text{Var}(m)} = 0,$$

$$\beta_{s,m+s} = \frac{\text{cov}(s,m+s)}{\text{Var}(m+s)} = \frac{\text{Var}(s)}{\text{Var}(m) + \text{Var}(s)} \geq 0.$$

$\beta_{s,m+s}$ is greater than zero unless $\text{Var}(s) = 0$, which is the special case of the risk-free asset. This points out that otherwise uncorrelated risky assets acquire beta in relation to the magnitude of their variance. Thus, relatively small risks that do not correlate with the rest of the market can be treated as near-zero-beta risks. A similar impact can be felt at the individual portfolio level where the inclusion of small (relative to the portfolio) orthogonal risks does not disturb the distribution of returns.

4.2 Diversification of Naturally Orthogonal Risks

Pure zero sum wagers may influence an individual's portfolio but do not impact the market portfolio. However, certain naturally orthogonal risks exist in the market even before a security interest is created by a wager. These risks can often be described as "insurance." Note that rain in Arizona is an uncertain event but does not, all of itself, impact the market. If, however, the rain does benefit or injure wealth, the benefit or the injury becomes an element of the market portfolio. Crop futures may reflect one form of "securitization by wager" applicable to weather uncertainty. Insurance represents another such securitization that usually applies only to an injurious outcome.

Insurance is usually characterized by two elements that may or may not apply to other risks: adverse selection and moral hazard. Dealing with these elements is invariably costly at the customer level and may also have an impact in insti-

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tutional transactions such as reinsurance. The cost of adverse selection becomes an insurance expense generally known as underwriting. Moral hazard gives rise to the expenses associated with claims settlement.

4.3 Demand Side Diversification

Consider n identical and independent[16] risks (e.g., home fire insurance). The individual acquires his or her individual risk along with home ownership. Absent adverse selection, moral hazard, and some administrative difficulties, each risk-averse homeowner would rather bear 1-nth of the total risk than all of his or her own. Suppose we could implement this risk sharing by trading shares of the individual risks in a perfect market. Then, as n grows large, the risk of each individual diminishes.[17] In the limit each individual may pay the expected loss discounted for the period at the risk-free rate.

In reality, the expenses of insurance require that risk-averse insureds must pay premiums in excess of the expected loss. The premium must be sufficient to cover all the expenses, and a viable market can exist only when there are n riskbearers who are sufficiently risk averse to pay the premiums (Pratt 1964). Further, in reality, the insurance and the enabling entity will be viable only when there is sufficient additional capital available to develop an arbitrarily high probability of solvency. Because of the operation of the law of large numbers, the amount of capital required per unit of insurance decreases although the total capital required is an increasing function of n. For small values of n, the cost of such capital is prohibitive to the insured, and, thus, we expect to observe large risk pools.

In a mutual insurance company, this additional capital is provided by the insureds as a portion of each premium. Capital is returned in the form of participating dividends. We will focus on the case of nonparticipating insurance issued by a stock insurance company. This requires that investors other than the insureds supply the capital necessary to ensure solvency.

What return must the capital providers expect to earn?

We have defined a case where insolvency is an insignificant issue. Capital will not be wiped out, and the suppliers will get a distribution of returns around a mean that is dictated by the mean losses. The central limit theorem allows us to conclude that this distribution will be normal, and thus the preconditions of the CAPM are satisfied. The CAPM, in turn, tells us that these insurance risks are inherently orthogonal, and, so long as they are small in comparison to the market, they will be priced as near-zero-beta securities.

What do the capital providers demand?

Contrary to the implications of the CAPM, providers demand substantial equity risk premiums, and insurance company managers price and reserve[18] their products accordingly. Even though the insurance industry is generally deemed to be competitive, the actuarial process adds "market value margins" into reserve calculations. These margins, decreased by losses in excess of the mean and increased by lesser losses, are released into reported earnings over time.

What are the sources of this dissonance between the CAPM and actuarial pricing?[19] What bearing does this have on the valuation of insurance risks embedded in the fair value of insurance liabilities?

4.4 Supply Side Diversification

We have seen that insurance diversifies demand side (customer) risks by pooling. But the providers of capital still require recompense for the residual risk of the pool despite the fact that this residual is naturally orthogonal to the rest of the capital markets.

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[16] Risks that are not independent may be diversified on the supply side (see next section).

[17] Interestingly, Karl Borch (1962) developed this theme in the context of a generalized reinsurance market. The logic and conclusions of his paper anticipated those of the CAPM developed two years later.

[18] "Reserve" is a verb describing the process by which actuarial liabilities (known as reserves) are estimated.

[19] William Sharpe (1991), in his Nobel Prize acceptance speech, addressed various practical impediments to the CAPM and observed that reductions in market frictions reduce these impediments over time. Better information and better transaction mechanisms may be expected to reduce market frictions.

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Insurance capital providers do not necessarily perceive the residual risk as orthogonal or price it at zero, even though they would generally agree that knowing the closing value of the S&P 500 tells them much about how GE may have traded and virtually nothing about how many cars crashed, homes burned down, or term insureds died.

In order for these risks to trade as near-zero-beta, they must be efficiently distributed over the entire market portfolio so that every diversified investor can hold minute portions. In addition, investors must have good information cheaply available. Innovative steps to improve trading efficiency and information have been taken in recent years.

Above we discussed risk reduction through pooling and noted that the residual risk (deviation from the mean by the entire risk pool) remained orthogonal but was not priced accordingly. We can also look at situations where risks that are naturally orthogonal to the rest of the capital markets are not themselves independent (e.g., home insurance in areas subject to widespread brush fires, hurricanes, or earthquakes). Because demand side diversification is not very effective in these situations, insurers have been looking elsewhere to lower costs.

An early effort led to reinsurance. Reinsurance, however, is primarily a further example of pooling. It can be effective when geographic subpools are independent of each other even though the each subpool contains highly correlated risks. Thus, a national reinsurer may cover homeowner policies in hurricane territories like Florida along with tornado territories that are further west and including earthquake and brush fire risks in California.

But some risks are simply too large for further pooling to be of much avail. Litzenberger, Beaglehole, and Reynolds (1996) and Cox and Pedersen (2000) show how catastrophe derivatives may eventually improve the pricing of lumpy, but naturally orthogonal, risks. The problem, as they see it, is that the risks are too concentrated within the insurance industry where “the big one” (Hurricane Andrew in 1993, had it hit the more populated eastern shore of Florida, or the Northridge earthquake in 1995, had the epicenter been in downtown Los Angeles) might wipe out 20% or even 50% of the entire industry capital base. Such an event might destroy as much as $100 billion of wealth. But the market portfolio can be estimated to exceed $25 trillion.

Suppose that the $100 billion loss can be represented by a Bernoulli distribution with an annual probability of occurrence, p, equal to 0.01. Then estimate the beta induced by including this risk in the market portfolio:

- The mean loss is $1 billion and could be priced accordingly if treated as an orthogonal risk. A smaller subclass of risk-averse investors[20] would undoubtedly demand a much higher premium to underwrite such a risk. What is the variance? The Bernoulli variance is $p(1 - p)$ times the square of the dollar loss:

$$\begin{aligned}\text{Var}(s) &= (0.01)(0.99)(10^{11\times 2})\\ &= 99 \times 10^{18}\$\text{-squared.}\end{aligned}$$

- The variance of the market portfolio can be estimated (assuming a 15% annual standard deviation) as

$$(0.15 \times 25 \times 10^{12})^2 = 14 \times 10^{24}\$\text{-squared.}$$

- Thus, beta[21] is approximately 7×10^{-6}.

In other words, if an efficient means were developed to distribute this risk widely, the expected loss plus expenses, without additive risk margins, could be discounted to compute the liability value. Such means include catastrophe bonds and derivatives as discussed by Litzenberger et al. and Cox and Pedersen. To date such securities have not been widely accepted by the capital markets, and returns required at issue, despite substantial collateral provisions, have been well in excess of risk-free rates. Future success, if any, of such mechanisms might encourage imitative developments of securities based on mortality, disability, personal property and other risks.

Although it would seem that orthogonal pricing (i.e., liabilities and premiums more or less free of market value margins for insurance risks) will not arrive soon, some explicit identification of these

[20] Such as the insurance industry with aggregate capital estimated to be a few hundred billion dollars.

[21] We have ignored the asymmetry of the Bernoulli distribution. Although this makes the specific beta calculation inaccurate, the conclusion that beta is very close to zero remains valid.

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margins within fair value accounting may be encouraged.[22] Separate identification (by adding risk charges to the numerator of any of the three valuation equations) would be valuable information that could increase price competition, provide statement users with better liability metrics, and hasten the development of securities designed to widen the trading of these risks. In turn, this better information and better trading devices can be anticipated to reduce the magnitude of charges associated with orthogonal insurance risks. Researchers would be well served by the disclosure of these risk margins no matter the progression of magnitudes.

5. LIQUIDITY RISK

In our discussion of valuation to this point we have not considered the impact of liquidity. Market liquidity can be defined as the ability of a market participant to transfer a risk, possibly at some future date, when preference for holding that risk has changed for whatever reasons.[23] A liquid security has a real option embedded in it.[24] The option is the ease with which the risk can be transferred to some other party. If liquidity is included in fair valuation, then essentially we are including the value of a real option. Note here that we are associating liquidity with the ability to transfer risk to another party. Liquidating a security is merely one of many ways to accomplish this; we are not presuming that a financial security needs to be sold to transfer risk. Transfer of risk can be done by purchasing an insurance or reinsurance contract or even by purchasing a security.

It is well known that illiquid securities command a higher yield in the marketplace than do their liquid counterparts. The lower yield of the liquid security is the price of the real option to transfer the risk. This has led some professionals to question whether to include some sort of liquidity recognition in the valuation of insurance liabilities. Some of the arguments for this are better than others. We find most of the arguments that have been put forth less persuasive than the real option consideration mentioned above.

For instance, it is argued that insurance liabilities are generally illiquid and long term in nature, and that this affords insurers the opportunity to invest in illiquid and/or longer-term securities, thereby capturing any available liquidity and/or term premium. Accordingly, advocates of this line of reasoning endorse the inclusion of a positive liquidity premium in the valuation of insurance liabilities. The effect of incorporating such a premium into the discount factor applied to the liabilities is to reduce their reported fair value.

We take issue with this line of reasoning. First, it is not the long-term or illiquid nature of insurance liabilities that allows insurers a comparative advantage at bearing liquidity risk. It is predictability of cash flows that allows a firm to take liquidity risk. To the extent that there are option-like features in the insurance liabilities, the insurer may get burned trying to take excessive liquidity risk in its assets.

Second, even if an insurer has liabilities that allow it to take liquidity risk, there may not be any available liquidity premium. If issuing long-term insurance liabilities means that the insurer can invest in longer-term securities, what if such securities do not command marginal liquidity premiums in the marketplace? Available evidence suggests the presence of liquidity premiums in the Treasury market for securities with maturities up to 2½ years, but the evidence is mixed for any incremental liquidity premium beyond that. Therefore, if a particular type of liability allows an insurer to extend its investment portfolio beyond 2½ years average maturity to, say, 5 years, it is not clear that it is picking up any additional expected return through its asset choice, on average. It certainly may gain higher yielding securities, but there is a big difference between yield and expected return. In the case of non-Treasury securities, higher yields at longer terms may be associated with greater credit or call risk, and not necessarily a liquidity premium.

In no case is it true that issuing illiquid liabilities really allows the issuer to invest in illiquid securities. It is cash flow predictability in the liabilities that allows a firm to take on liquidity

[22] As noted earlier, it took several years before the MBS markets substantially reduced the required spreads for noise.

[23] Our treatment in this section relies heavily on an American Academy of Actuaries Valuation Task Force–sponsored discussion between Jeremy Gold, Luke Girard, and Marsha Wallace.

[24] Amram and Kulatilaka (1998) provide a survey of the literature on real options.

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risk in its assets. Issuing nonredeemable (neither putable nor surrenderable) liabilities is what is needed if a company wishes to invest illiquidly and capture any available liquidity premium. If liquidity is not accepted as a factor in fair valuation, then exit value will tend to overstate fair value systematically.[25]

If an insurance company is in the business of accepting mortality or morbidity risk, it will do so. However, at some point it may decide to get out, perhaps because it wants to be an annuity company: that is, its preference for mortality risk has changed. To do this it must find a suitable buyer of the risk. The insurance company may find that the number of potential buyers is limited. This may be because (1) the nature of the risk itself may not attract buyers, (2) there is not sufficient experience to evaluate the risk, (3) the risk is complex, (4) the amount of the risk is too small or too large, (5) regulatory issues may impede the transfer of the risk, or (6) the information is asymmetric in favor of the seller, among other reasons. We frequently see similar issues on the asset side with private placements, real estate, mortgage loans, and many structured securities. We also observe it in the public securities markets. And liquidity is dynamic. Sometimes it is very good, and sometimes, as it was in the autumn of 1998, it is very bad.

Although it is true that the market will price illiquidity, after the securities have been issued illiquidity is not an aspect of the indebtedness, per se. In neither the public nor the private placement market can the issuer be forced to prepay. In this sense a bullet GIC is no more illiquid to the issuer than is a zero coupon bond. An outstanding GIC may be priced by its holder at a lower value than the corresponding tradable zero coupon bond because the GIC is less liquid, but the liability value of each (assuming that the credit standing of each is the same and ignoring any interest that the issuer may have in prepaying) should be identical. After issuance, the illiquidity premium seems to be an aspect of ownership rather than issuership. The lender has assumed the risk, and if it is an illiquid risk, the lender is restricted by the market from transferring the risk.

We are left with the issue of taking into account liquidity in fair valuation due to the real option afforded to the issuer of the liability to transfer it off its books. It is difficult to say that an estimate of the possible exit price is a proper measure of the liability value. The reason for this is that the liabilities do not trade in an open, transparent market; rather, there is a small, ephemeral market with only a few players, and transfer prices are negotiated privately. Presumably the purchaser of the risk is going to assume it only if it has a positive net present value (NPV). Similarly, the originator of the insurance contract will also sell the insurance only if it has a positive NPV, as most or all of the ex ante NPV will accrue on the liability side, not the asset side, of an insurer's balance sheet. Therefore, we think that the inclusion of a liquidity risk adjustment on the insurance liabilities is problematic if the notion of exit price is to be used.

6. DECOMPOSITION AND TRANSPARENCY

A key benefit of the decomposition approach is that it increases transparency. Insurance liabilities are far more complex than corporate bonds. Any reasonably competent analyst, given a market price and the details of a corporate bond (coupon rate and maturity date), could use Treasury bond data to figure out the synthetic Treasury value of the corporate bond and the value of the put option. The put option is just the difference between the synthetic Treasury value of the bond and the market price of the bond. The relative ease of this decomposition is due to the limited information required to describe fully the cash flows of a corporate bond. Thus, for a corporate bond, it is fully adequate to report only its fair value on a net basis, where the default put is netted against the default-free equivalent security. The relative impact of interest rate changes and credit quality changes is easy to discern. Similarly, for a GIC it might be adequate simply to report the market (fair) value of the liability, provided that sufficient information is given to allow the analyst to compute the default-free

[25] A more subtle argument is that if an insurer issues policies to a "stodgy clientele," the surrender/lapse option will be utilized less efficiently against the insurer, garnering additional value to the insurer. Although this may be true, it is already captured in the stochastic interest rate valuation model, because the lapse function is explicitly incorporated into the cash flow component of the model. Therefore, to include it again would be redundant.

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equivalent GIC. For more complex insurance liabilities the decomposition approach has advantages.

In the case of more complex insurance liabilities, the increased transparency of the decomposition approach is valuable for analysts, regulators, investors, and management. Analysts would be able to compare the structure of liabilities from one company to another more easily because of the consistent use of Treasury rate lattices or paths in calculating the risk-free present value of interest-rate-contingent liability cash flows (the synthetic Treasury value of the bond). Then a contra-liability (the put option) would summarize the condition of the company backing the liabilities. If the liabilities were to be transferred from one company to another, the contra-liability might change, but not the present value of liability cash flows. This would aid in mergers and acquisitions analysis as well as facilitate decision making about sales of a block of business. Regulators would also benefit from this decomposition. The synthetic Treasury value of liabilities is like a defeasance value of the liabilities.[26] The put option value captures the risk inherent in the company backing the liabilities. Similar reasoning applies to investors and managers who are concerned with the condition of the company.

The put option value is relatively easy to compute. The same possible interest-rate-contingent cash flows that are discounted using Treasury rate lattices or paths to arrive at the synthetic Treasury portion of the decomposed liability value can also be discounted at risky interest rates. A spread, with appropriate maturity and risk dependencies, can be added to the Treasury interest rates to discount the projected liabilities. The difference between the two present values is the value of the put option. Although it might seem that it would be easier just to discount with a spread and call that the fair value, the decomposition is valuable for the reasons listed above.

In the theoretical derivation of Equation (4) we assume that the only risk is default risk. In practice the put option value in Equation (4) must include at least default risk, liquidity risk, and potentially orthogonal risks such as underwriting risk, expense risk, and mortality or casualty risk. The problem with reporting a single number as the fair value of liabilities is captured by an observation by attorney Johnny Cochrane. He asks, "What do you do if you are told that a cockroach has walked across your plate of spaghetti? You throw the whole thing away because you don't know which pieces of food the roach stepped on." A single netted number reported as the fair value of liabilities contains many assumptions and complex calculations. These assumptions and calculations are far more complex than for a simple bond or loan. The analyst looking at the report will be tempted to throw out the fair value number, just as Cochrane did with his spaghetti, if the analyst can't see the intermediate steps that lead to the estimate of fair value. The decomposition makes for greater transparency as the analyst can see where the assumptions affect the interest-rate-contingent synthetic Treasury value of the liability and where the assumptions affect the other risk portions of the fair value of the liability.

7. Concluding Comments

There is much work still to be done to extend the reasoning in this paper to more complex liabilities. In fact, it may well be that the best we can do at this point is to estimate future possible cash flows with their interest rate contingencies and discount them by Treasury interest rate processes and then by interest rate processes that incorporate appropriate spreads. In this way we can estimate the pieces of the decomposed value of insurance liabilities, and we know exactly on which piece Cochrane's cockroach has tread.

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[26] In this context "defeasance value" means the value of a portfolio of Treasury securities and derivatives that fully funds the expected cash flows, including interest rate contingencies, of the insurance liabilities being considered.

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